AP1 Momentum

1. A cart full of water travels horizontally on a frictionless track with initial velocity **v**. As shown in the diagram, in the back wall of the cart there is a small opening near the bottom of the wall that allows water to stream out. Considering just the cart itself (and not the water inside it), which of the following most accurately describes the characteristics of the cart?
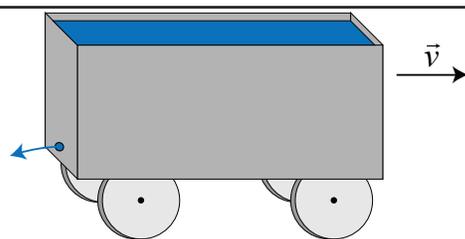
	Speed	**Kinetic Energy**
(A)	stays the same	stays the same
(B)	increases	increases
(C)	stays the same	increases
(D)	increases	stays the same

Answer: B

As the water streams out of the cart, the water is pushed out of the cart by pressure from above, exerting a reactionary force pushing the cart and its contents forward, increasing the speed of the cart. As the speed of the cart is increasing, and its mass remains the same, the kinetic energy of the cart must also increase.

EK: 3.A.4 If one object exerts a force on a second object, the second object always exerts a force of equal magnitude on the first object in the opposite direction. 3.E.1 The change in the kinetic energy of an object depends on the force exerted on the object and on the displacement of the object during the interval that the force is exerted. 5.A.2 For all systems under all circumstances, energy, charge, linear momentum, and angular momentum are conserved. For an isolated or a closed system, conserved quantities are constant. An open system is one that exchanges any conserved quantity with its surroundings.

SP: 6.4 The student can make claims and predictions about natural phenomena based on scientific theories and models. 7.2 The student can connect concepts in and across domain(s) to generalize or extrapolate in and/or across enduring understandings and/or big ideas.

LO: 3.A.4.1 The student is able to construct explanations of physical situations involving the interaction of bodies using Newton's third law and the representation of action-reaction pairs of forces. 3.E.1.1 The student is able to make predictions about the changes in kinetic energy of an object based on considerations of the direction of the net force on the object as the object moves. 5.A.2.1 The student is able to define open and closed systems for everyday situations and apply conservation concepts for energy, charge, and linear momentum to those situations.

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2. A firecracker is launched with an initial velocity of 70 m/s at an angle of 73° with the horizontal. The firecracker explodes at its highest point, splitting into three equal pieces. One piece continues at its same horizontal speed, but moves vertically upward at 10 m/s immediately after the explosion. A second piece moves vertically downward at 10 m/s, but with a horizontal velocity of 30 m/s backward immediately after the explosion. Determine the speed of the remaining piece of the firecracker immediately following the explosion. Neglect air resistance.
(A) 0 m/s
(B) 10 m/s
(C) 70 m/s
(D) 90 m/s

Answer: (C) 70 m/s

You can approach this problem as either a center-of-mass problem with no external forces, or a conservation of linear momentum problem, again with no external forces. The horizontal velocity of the intact firecracker at its highest point is 20 m/s. Once the firecracker explodes, three pieces of equal mass are ejected. The vertical components of the velocity of the first and second pieces are equal in magnitude and opposite in direction, so the vertical component of velocity for the remaining piece must be zero. The horizontal component of the first piece of the firecracker has no change in horizontal velocity. Therefore, the change in horizontal velocity of the second piece must balance the change in horizontal velocity of the third piece. If the second piece is moving backward at 30 m/s (a change in velocity of -50 m/s), the third piece must experience a change in velocity of +50 m/s, for a total velocity of 70 m/s horizontally, and a final speed of 70 m/s.

EK: 4.A.1 The linear motion of a system can be described by the displacement, velocity, and acceleration of its center of mass. 5.A.2 For all systems under all circumstances, linear momentum is conserved. For an isolated or a closed system, conserved quantities are constant. An open system is one that exchanges any conserved quantity with its surroundings. 5.D.3 The velocity of the center of mass of the system cannot be changed by an interaction within the system.

SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 2.2 The student can apply mathematical routines to quantities that describe natural phenomena. 7.2 The student can connect concepts in and across domain(s) to generalize or extrapolate in and/or across enduring understandings and/or big ideas.

LO: 4.A.1.1 The student is able to use representations of the center of mass of an isolated two-object system to analyze the motion of the system qualitatively and semiquantitatively. 5.A.2.1 The student is able to define open and closed systems for everyday situations and apply conservation concepts for energy, charge, and linear momentum to those situations. 5.D.3.1 The student is able to predict the velocity of the center of mass of a system when there is no interaction outside of the system but there is an interaction within the system (i.e., the student simply recognizes that interactions within a system do not affect the center of mass motion of the system and is able to determine that there is no external force).

3. A cart traveling on a smooth track with velocity v collides and sticks to an identical cart on the track, initially at rest. What is the maximum percentage of the cart's initial kinetic energy maintained as kinetic energy in the two-cart system?

(A) 25%
(B) 50%
(C) 75%
(D) 100%

Answer: (B) 50%

Consistent with the law of conservation of momentum, following the collision the maximum possible speed of the two carts combined is v/2. Kinetic Energy, however, is related to the mass and the square of the velocity. Following the collision, the kinetic energy of the combined carts doubles due to the doubling of mass, but is quartered due to the effect of $(v/2)^2$. The product of these two effects, then, is an effective reduction in the kinetic energy of the combined carts by at least 50%. The missing energy must have been lost through non-conservative means such as heat and sound.

EK: 4.C.2 Mechanical energy (the sum of kinetic and potential energy) is transferred into or out of a system when an external force is exerted on a system such that a component of the force is parallel to its displacement. The process through which the energy is transferred is called work. 5.D.2 In a collision between objects, linear momentum is conserved. In an inelastic collision, kinetic energy is not the same before and after the collision.

SP: 6.4 The student can make claims and predictions about natural phenomena based on scientific theories and models. 7.2 The student can connect concepts in and across domain(s) to generalize or extrapolate in and/or across enduring understandings and/or big ideas.

LO: 4.C.2.2 The student is able to apply the concepts of Conservation of Energy and the Work-Energy theorem to determine qualitatively and/or quantitatively that work done on a two-object system in linear motion will change the kinetic energy of the center of mass of the system, the potential energy of the systems, and/or the internal energy of the system. 5.D.2.3 The student is able to apply the conservation of linear momentum to a closed system of objects involved in an inelastic collision to predict the change in kinetic energy.

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4. A identical bullets are fired into identical wood blocks in two different positions as shown in the diagram at right. In each case the wood block (with the bullet embedded) rises to a certain height before returning to the surface of the Earth. Assuming the bullets have the same initial velocities, which block will go higher, and why?
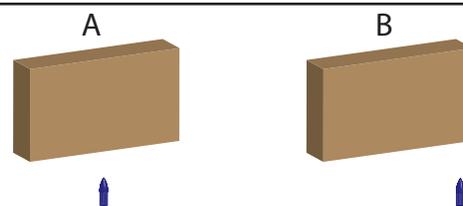
(A) Block A will go higher since all of the bullet's initial kinetic energy is transformed into gravitational potential energy (height) in scenario A, while some of the bullet's initial kinetic energy in scenario B is transformed into rotational kinetic energy, therefore B doesn't go as high.
(B) Block B will go higher since the rotational kinetic energy in situation B adds to the translational kinetic energy imparted by the bullet, while Block A doesn't receive this additional rotational kinetic energy.
(C) The blocks will reach the same maximum height due to conservation of linear momentum.
(D) The blocks will reach the same maximum height due to conservation of angular momentum.

Answer: (C) The blocks will reach the same maximum height due to conservation of linear momentum. It is true that block B will also have some amount of rotational kinetic energy, and both blocks have the same maximum gravitational potential energy. This difference in energy is accounted for in realizing this is an inelastic collisions, and the difference in energy of the blocks is due to differences in penetration of the bullets into the blocks.

Note that this experiment, as well as an explanation of results, is available from Derek Muller's Veritasium YouTube Channel, and is a highly recommended follow-up exercise to this problem: https://www.youtube.com/watch?v=vWVZ6APXM4w

EK: 5.A.2 For all systems under all circumstances, energy, charge, linear momentum, and angular momentum are conserved. For an isolated or a closed system, conserved quantities are constant. An open system is one that exchanges any conserved quantity with its surroundings. 5.D.2 In a collision between objects, linear momentum is conserved. In an inelastic collision, kinetic energy is not the same before and after the collision.

SP: 1.5 The student can reexpress key elements of natural phenomena across multiple representations in the domain. 6.4 The student can make claims and predictions about natural phenomena based on scientific theories and models. 6.5 The student can evaluate alternative scientific explanations. 7.2 The student can connect concepts in and across domain(s) to generalize or extrapolate in and/or across enduring understandings and/or big ideas.

LO: 5.A.2.1 The student is able to define open and closed systems for everyday situations and apply conservation concepts for energy, charge, and linear momentum to those situations. 5.D.2.3 The student is able to apply the conservation of linear momentum to a closed system of objects involved in an inelastic collision to predict the change in kinetic energy.

Difficulty: 2

AP1 Momentum

5. Two small, uniform balls of identical density and size are fired from a toy gun toward a wooden block. Ball A is highly elastic and bounces backward after striking the block. Ball B is made of clay and sticks to the wooden block upon impact. Which of the following statements best describes the effects of the collision with the block?
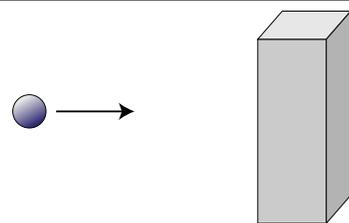
(A)   Ball A has a greater change in momentum and a greater change in kinetic energy than ball B.
(B)   Ball A has a greater change in momentum but a smaller change in kinetic energy than ball B.
(C)   Ball A has a smaller change in momentum and a smaller change in kinetic energy than ball B.
(D)   Ball A has a smaller change in momentum but a greater change in kinetic energy than ball B

Answer: (B) Ball A has a greater change in momentum but a smaller change in kinetic energy than ball B.

Both balls have the same initial momentum prior to striking the block. Following the collision, however, the elastic ball, Ball A, bounces backward, transferring up to twice its initial momentum to the block through the larger impulse. Ball B, however, sticks to the block, transferring its initial momentum to the block through an impulse equal to its initial momentum. Therefore, Ball A transfers more momentum to the block.

With respect to kinetic energy, however, the story changes. Ball A maintains some kinetic energy as it rebounds off the block, whereas Ball B loses all its kinetic energy as it comes to rest. Therefore, Ball A has a smaller change in kinetic energy than ball B.

EK: 4.C.2 Mechanical energy (the sum of kinetic and potential energy) is transferred into or out of a system when an external force is exerted on a system such that a component of the force is parallel to its displacement. The process through which the energy is transferred is called work. 5.D.2 In a collision between objects, linear momentum is conserved. In an inelastic collision, kinetic energy is not the same before and after the collision.

SP: 6.4 The student can make claims and predictions about natural phenomena based on scientific theories and models. 7.2 The student can connect concepts in and across domain(s) to generalize or extrapolate in and/or across enduring understandings and/or big ideas.

LO: 4.C.2.2 The student is able to apply the concepts of Conservation of Energy and the Work-Energy theorem to determine qualitatively and/or quantitatively that work done on a two-object system in linear motion will change the kinetic energy of the center of mass of the system, the potential energy of the systems, and/or the internal energy of the system. 5.D.2.1 The student is able to qualitatively predict, in terms of linear momentum and kinetic energy, how the outcome of a collision between two objects changes depending on whether the collision is elastic or inelastic.

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6. An 80 kg student stands on the left end of a 240-kg log which is floating in the water (which you may treat as a frictionless surface). The student and the log are both initially at rest.
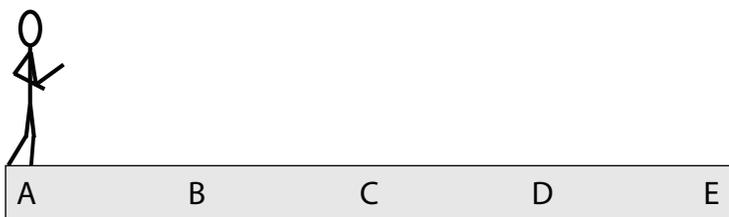
(a) Which point is the approximate center of mass of the student-log system?

(b) The student walks to the far end of the log at a constant speed of 2 m/s. As the student walks to the right, describe the motion of the log. Provide quantitative values wherever possible.

(c) What is the velocity of the center of mass of the student-log system while the student is walking?

Answer:
(a) (B) is the approximate center of mass of the system.
(b) The log moves to the left with a speed of 0.67 m/s as the student moves right.
(c) The velocity of the center of mass of the student-log system is 0 m/s while the student is walking since the initial velocity of the center of mass of the system was 0 m/s and no external forces were applied.

EK: 5.D.1 In a collision between objects, linear momentum is conserved. In an elastic collision, kinetic energy is the same before and after. 5.D.3 The velocity of the center of mass of the system cannot be changed by an interaction within the system.

SP: 2.2 The student can apply mathematical routines to quantities that describe natural phenomena.

LO: 5.D.1.1 The student is able to make qualitative predictions about natural phenomena based on conservation of linear momentum and restoration of kinetic energy in elastic collisions. 5.D.3.1 The student is able to predict the velocity of the center of mass of a system when there is no interaction outside of the system but there is an interaction within the system (i.e., the student simply recognizes that interactions within a system do not affect the center of mass motion of the system and is able to determine that there is no external force).

Difficulty: 1

AP1 Momentum

7. A 0.5-kilogram block slides at 20 m/s on a smooth frictionless surface toward a stationary sphere, shown below left.

20 m/s

0.5 kg
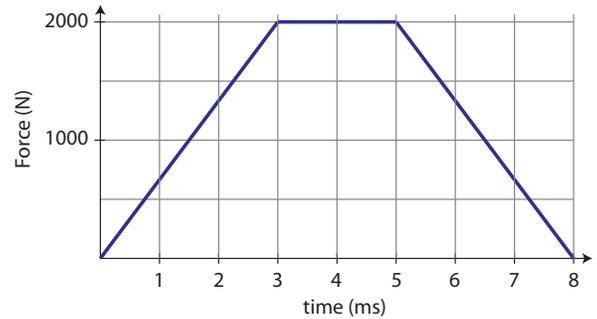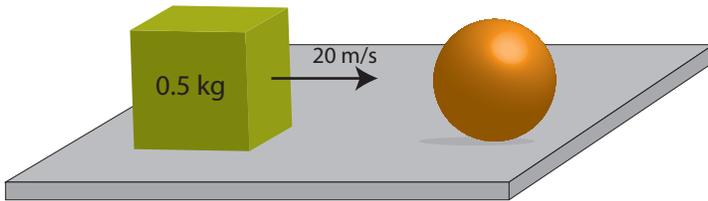
The sphere is half the volume of the block, but is eight times as dense. The block strikes the sphere at time t=0. A plot of the force exerted on the cube by the ball as a function of time is shown above right.

(a) What is the impulse applied to the block?

(b) What is the speed of the ball immediately following the collision?

(c) What is the velocity of the cube immediately following the collision? (State both direction and magnitude.)

(d) Is this an elastic collision? Justify your answer.

Answer:

   (a) The impulse is the area under the curve which is 10 N•s.

   (b) The impulse applied is equal to the change of momentum, therefore

$$J = \Delta p = m\Delta v \rightarrow \Delta v = \frac{J}{m} = \frac{10 N \bullet s}{2kg} = 5 \, m/_s$$

   (c) The change in velocity of the block can also be found from the impulse-momentum theorem:

$$J = \Delta p = m\Delta v \rightarrow \Delta v = \frac{J}{m} = \frac{-10 N \bullet s}{0.5kg} = -20 \, m/_s$$

       The velocity of the block after the collision is therefore 0 m/s, since it was going 20 m/s, and its change of velocity is -20 m/s. The direction is unnecessary -- the block is stationary.

   (d) This is an inelastic collision since kinetic energy is **not** conserved. The initial kinetic energy of the block-sphere system is 100 J, and the final kinetic energy of the block-sphere system is 25 J.

EK: 1.E.1 Matter has a property called density. 3.D.2 The change in momentum of an object occurs over a time interval. 5.D.1 In a collision between objects, linear momentum is conserved. In an elastic collision, kinetic energy is the same before and after.

SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 2.2 The student can apply mathematical routines to quantities that describe natural phenomena. 5.3 The student can evaluate the evidence provided by data sets in relation to a particular scientific question. 6.1 The student can justify claims with evidence. 7.2 The student can connect concepts in and across domain(s) to generalize or extrapolate in and/or across enduring understandings and/or big ideas.

LO: 1.E.1.1 The student is able to predict the densities, differences in densities, or changes in densities under different conditions for natural phenomena and design an investigation to verify the prediction. 3.D.2.3 The student is able to analyze data to characterize the change in momentum of an object from the average force exerted on the object and the interval of time during which the force is exerted. 5.D.1.1 The student is able to make qualitative predictions about natural phenomena based on conservation of linear momentum and restoration of kinetic energy in elastic collisions. 5.D.1.5 The student is able to classify a given collision situation as elastic or inelastic, justify the selection of conservation of linear momentum and restoration of kinetic energy as the appropriate principles for analyzing an elastic collision, solve for missing variables, and calculate their values.

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8. A proton (mass=m) and a lithium nucleus (mass=7m) undergo an elastic collision as shown below.
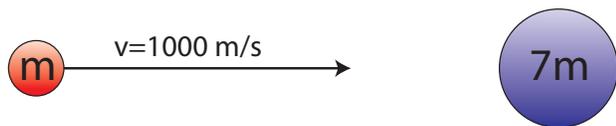
Find the velocity of the lithium nucleus following the collision.

Answer: 250 m/s

First, solve for the velocity of the proton as a function of the velocity of the lithium nucleus using conservation of linear momentum.

$$\vec{p}_b = \vec{p}_a \rightarrow 1000m = mv_p = 7mv_L \rightarrow v_p = 1000 - 7v_L$$

Next, utilize conservation of kinetic energy since this is an elastic collision to solve for the velocity of the lithium nucleus.

$$K_b = K_a \rightarrow \tfrac{1}{2}m(1000)^2 = \tfrac{1}{2}mv_p^2 + \tfrac{1}{2}(7m)v_L^2 \rightarrow 10^6 = v_p^2 + 7v_L^2 = (1000 - 7v_L)^2 + 7v_L^2 \rightarrow$$
$$v_L = 250 \, m/s$$

EK: 5.D.1 In a collision between objects, linear momentum is conserved. In an elastic collision, kinetic energy is the same before and after.

SP: 2.2 The student can apply mathematical routines to quantities that describe natural phenomena. 5.3 The student can evaluate the evidence provided by data sets in relation to a particular scientific question.

LO: 5.D.1.3 The student is able to apply mathematical routines appropriately to problems involving elastic collisions in one dimension and justify the selection of those mathematical routines based on conservation of momentum and restoration of kinetic energy. 5.D.1.5 The student is able to classify a given collision situation as elastic or inelastic, justify the selection of conservation of linear momentum and restoration of kinetic energy as the appropriate principles for analyzing an elastic collision, solve for missing variables, and calculate their values.

Difficulty: 3